Mathematical investigation (2)

Investigating is a great way to learn to think mathematically, apply logic, spot patterns and improve our perseverance.

What are the chances of that?!

Aims:

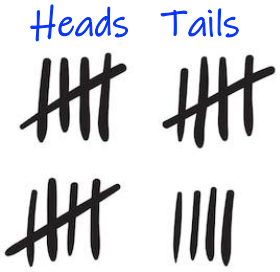
- Apply learning about **fractions** to explore **probability** then use this knowledge in a game.

You will need:

- Two coins

How many heads and how many tails do you predict if you toss a coin 30 times?

- Try it and see.
- Use tally marks to record the results for numbers of heads and tails so that you don't lose track, e.g.



Did you **predict** half tails and half heads?

Was the result close to 15 heads and 15 tails?

*A result of 12/18, 13/17 or 14/16 is quite close to the prediction. A result of 2 or 3 heads and 27 or 28 tails is quite **unlikely**!*

Now get a second coin. You will toss the **two coins together** 60 times.

First, **predict** how many times you will get:

a) two heads

b) two tails

c) one of each

My prediction
Two heads:
Two tails:
One of each:

Did you predict a third (20) for each result? Is that what has happened?

To understand what's going on, we need to look at each coin in turn…

It is probably easier to understand if we use two different coins (e.g. 10p & 2p).

There are **four** possible results:

2p **heads**, 10p **heads** (we could give this a code, e.g. **HH**)

2p **tails**, 10p **tails** (code = **TT**)

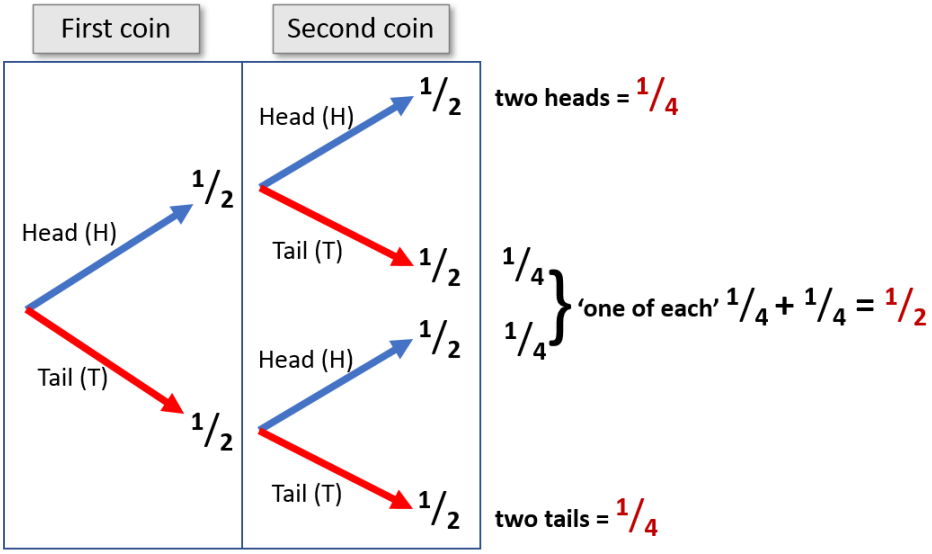
2p **heads**, 10p **tails** (code = **HT**)

10p **heads**, 2p **tails** (code = **HT**)

So, we are likely to get twice as many 'one of each' (HT) as 'two heads' (HH) or 'two tails' (TT)!

Looking at the results above, we see that there is only a 1 in 4 ($^1/_4$) chance of getting two heads, i.e. **HH** only appears as one of the four possible outcomes.

Another way to reach the answer $^1/_4$ is to think about the fractions involved. Let's draw a picture to see this clearly…



So, the chance of following *any* one pathway (HH, HT, TH or TT) is $^1/_4$. Following the pathway to get **HH**, the chance of getting a head on the first throw is 1 in 2 or $^1/_2$. On the second toss, the chance of getting a head is also $^1/_2$, **but** it's $^1/_2$ of the first $^1/_2$, or $^1/_2$ x $^1/_2$, which = $^1/_4$.

If we tossed 3 coins, what would be the chance of getting 3 heads?

Chancy Dice

You will need:

- Two 1-6 dice

Let's apply what we've learnt about coins to dice.

- If I throw one dice, there is an equal chance of getting a 1, 2, 3, 4, 5 or 6.
- So, I have a 1 in 6 chance of scoring 4; the same chance of scoring 3, etc.
- What happens with 2 dice? What is the chance of scoring 4? Or 10…?

What is the lowest and highest possible total with two dice?

We can score 2 (1 + 1) or 12 (6 + 6).

So, anything from 2 to 12 is possible, but are all the scores equally likely? What do you predict?

Roll the pair of dice 100 times and create a tally chart of the results.

Score	Tally to show **Number of times scored** (**frequency**)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

Do you notice any patterns emerging?

Why is the score more likely to be 6, 7 or 8 than 2 or 12…?

Hint!

How many pairs of dice throws give a total of 2 or 12?
How many give a total of 6, 7 or 8?
List the possibilities as we did for the coins…

Now you understand the chances! You can win this game!

Crossing the river

You will need:

- Two 1-6 dice
- 20 counters or coins *(provided to cut out if required)*
- The board provided below

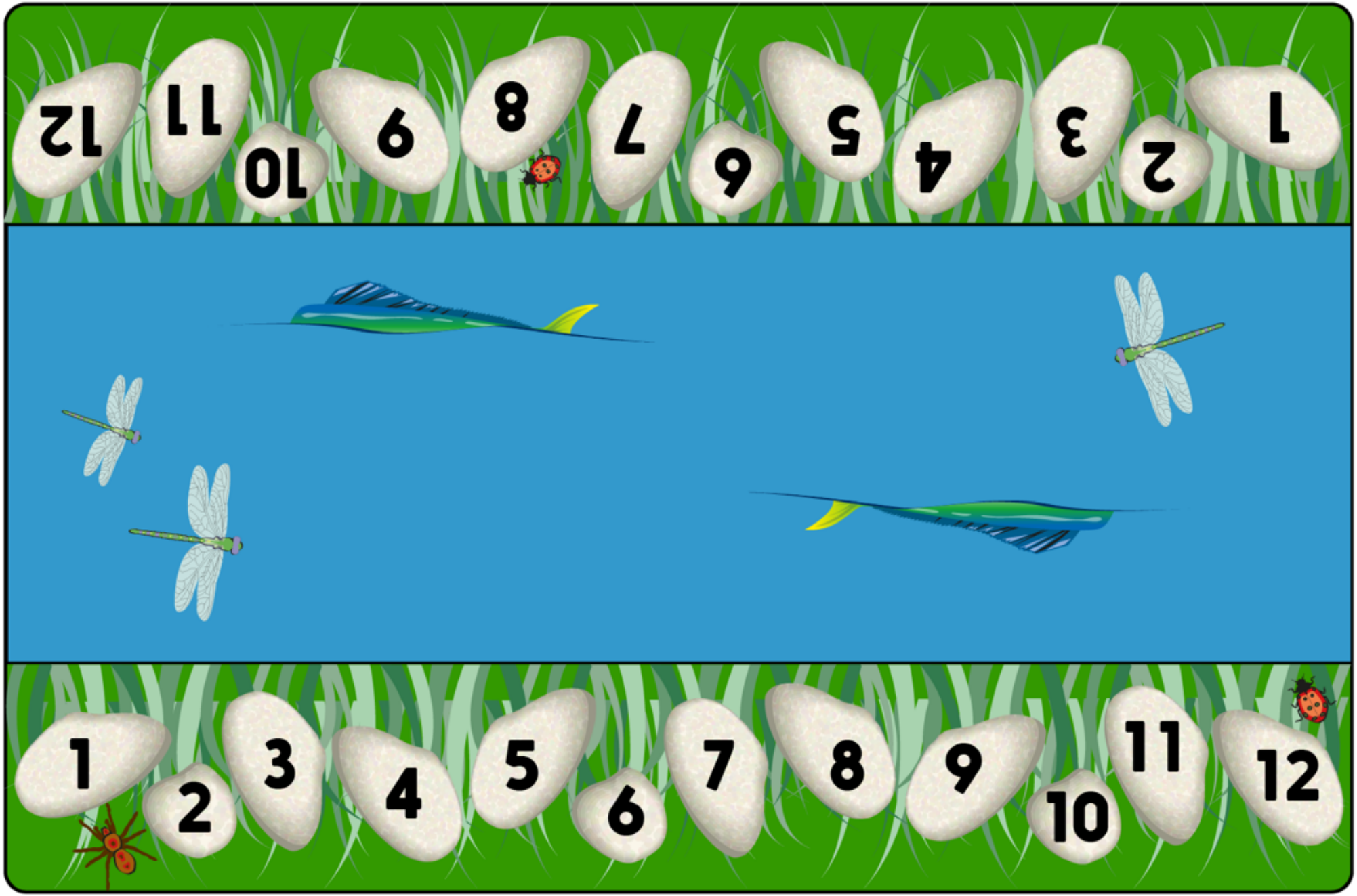
This is a game for two players who should place the board between them. Each player starts with **10 counters**.

- Place your counters on your side of the river in ***any*** of the number boxes.
  You can put all 10 in different places or several in one or two of the boxes, as you prefer.
- Take turns to throw the dice; add to find the total. If you have a counter in that box it can 'cross the river' and leave the board.
- The winner is the first person to get all 10 counters across the river.

Try the game several times.

- What is a **good strategy?** Think about what happened when we tallied the results of 100 dice throws.
- What are the **best numbers** to put your counters on?
- Should you put **all** your counters on those numbers?
- Are there any numbers you would advise against putting *any* counters on?
- What if you started with 20 rather than 10 counters, would you stick to the same strategy?

Crossing the river: game board



Counters to cut out